On Numerical Methods for Radiative Heat Transfer - Repetition

by

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• Radiation without participating media
  » Surface to surface radiation (S2S)
    > Surface phenomena
    > No change in radiation intensity
    > Basic radiation quantity – emissive power
    > Analysis simple
    > Net exchange based on Configuration Factors (shape or view factors)
Configuration-View-Shape factors

\[ F_{12} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} \, dA_1 \, dA_2 \]
Exchange Non-Black Surfaces (S2S)

\[ \dot{Q}_i = A_i (J_i - G_i) \]

\[ J_i = \varepsilon_i E_{B,i} + \rho_i G_i \]

\[ \rho_i = 1 - \varepsilon_i \]

\[ \dot{Q}_i = A_i \frac{\varepsilon_i}{1 - \varepsilon_i} (E_{B,i} - J_i) \]

\[ \dot{Q}_i = A_i \sum_k F_{ik} (J_i - J_k) \]
Governing equations - CFD

\[
\frac{\partial (\rho \phi)}{\partial t} + \frac{\partial (\rho U_j \phi)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_\phi \frac{\partial \phi}{\partial x_j} \right) + S_\phi
\]

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \Gamma_\phi )</th>
<th>( S_\phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{1} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( U_j )</td>
<td>( \mu + \mu_i )</td>
<td>(- \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu + \mu_i \right) \left( \frac{\partial U_j}{\partial x_i} - \frac{2}{3} \frac{\partial U_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \rho k \delta_{ij} )</td>
</tr>
<tr>
<td>( h )</td>
<td>( \mu / Pr + \mu_i / Pr_i )</td>
<td>(- \frac{\partial P}{\partial t} - \frac{\partial q_{R,j}}{\partial x_j} )</td>
</tr>
<tr>
<td>( Y_\alpha )</td>
<td>( \mu / Sc + \mu_i / Sc_i )</td>
<td>( \dot{\omega}_\alpha )</td>
</tr>
<tr>
<td>( k )</td>
<td>( \mu + \mu_i / \sigma_k )</td>
<td>( P_k - \rho \varepsilon )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>( \mu + \mu_i / \sigma_\varepsilon )</td>
<td>( f_1 C_{1e} \frac{\varepsilon}{k} P_k - \rho \varepsilon f_2 C_{2e} \frac{\varepsilon}{k} )</td>
</tr>
</tbody>
</table>

\[ \mu_i = \rho f_{\mu} C_{\mu} k^2 / \varepsilon \]
Detailed Analysis of Radiative Exchange in Participating Media

• Overall aim is to enable accurate heat load calculations
• Key is the so-called radiative transfer equation (RTE), plus
• Determine absorption coefficients for gases
• Determine scattering coefficients of particles
Radiative Transfer Equation, RTE (omitted transient term)

\[
\frac{dI_\lambda}{ds} = \hat{s} \cdot \nabla I_\lambda = \kappa_\lambda I_{b\lambda} - \beta_\lambda I_\lambda + \frac{\sigma_{s\lambda}}{4\pi} \int I_\lambda(\hat{s}_i) \Phi_\lambda(\hat{s}_i, \hat{s}) d\Omega_i
\]

\[
\beta_\lambda = \kappa_\lambda + \sigma_{s\lambda}
\]
Divergence of radiative heat flux

\[ \nabla \cdot \int_{4\pi} I_{\lambda} \hat{s} d\Omega = 4\pi \kappa_{\lambda} I_{b\lambda} - \int_{4\pi} \beta_{\lambda} I_{\lambda} (\hat{s}) d\Omega + \frac{\sigma_{s\lambda}}{4\pi} \int_{4\pi} I_{\lambda} (\hat{s}_i) \left( \int_{4\pi} \Phi_{\lambda} (\hat{s}_i, \hat{s}) d\Omega \right) d\Omega_i \]

\[ \mathbf{q}_{\lambda} = \int_{4\pi} I_{\lambda} \hat{s} d\Omega \]

\[ \int_{4\pi} \Phi_{\lambda} (\hat{s}_i, \hat{s}) d\Omega = 4\pi \]
RTE coupling with Energy Equation

\[ S_R = \text{div} \bar{q}_R = \kappa \left( 4\sigma n^2 T^4 - G \right) \]

Energy equation

Species equations

NS-equations

\[ G_\lambda = \int_{4\pi} I_\lambda d\Omega \]
Solution methods for the radiative transfer equation, RTE

1) Spherical Harmonics Method - also called the $P_N$ – approximation method
2) Discrete Ordinates Method (DOM)
3) Discrete Transfer Method (DTM)
4) Finite Volume Method (FVM)
5) Zonal Method
6) Monte Carlo Method (MC)
The $P_1$ radiation model

• The $P_1$, model solves an advection-diffusion equation for the mean local incident radiation (irradiance) $G$

• Consequently the gradient of the radiation flux can be directly substituted into the energy equation to account for heat sources or sinks due to radiation

\[
\nabla \cdot \left( \frac{1}{3\beta - A_1\sigma_s} \nabla G \right) = -\kappa(4n^2\sigma T^4 - G)
\]
RTE, radiative properties

Gas properties
- LBL (line-by-line)
- NBM
- EWBM (exponential wide band model)
- Global models

Particle properties (soot)
- Rayleigh
- Mie
Line overlapping and broadening

24 atm, 1.704 m path

$\text{H}_2\text{O, CO}_2, \text{CO}$
Interaction of an electromagnetic wave with a particle

![Diagram showing interactions](image)

$x = \frac{2\pi a}{\lambda}$

Introduce
Interaction of an electromagnetic wave with a particle

$X \ll 1$ Rayleigh scattering
$X \sim 1$ Mie scattering
$X \gg 1$ geometric optics applicable

\[ x = \frac{2\pi a}{\lambda} \]
Radiation properties of soot

- Scattering of soot
  
  Main difference between particle and gas:

  - Mie theory
    
    Suitable for optical large particles

  - Rayleigh theory
    
    Suitable for optical small particles
Small soot particles

• Scattering is negligible

\[ \kappa_\lambda = \frac{C \cdot f_v}{\lambda^a} \]

*C and a are empirical constants*
Type of Particles

Analysis of ash samples from cyclones and Electro-Static Precipitator (ESP) in a grate biomass boiler, fueled by bark and wood chips, shows:

• The fly-ash and char are the main particles, and

• The ratio of fly-ash and char to total amount of ash is 0.72 and 0.28, respectively.
Particle Compositions

- Sub-micron particles (< 1 μm) are mainly composed of C (in form of soot), S, Cl and heavy metals.

- Super-micron particles:
  - Fly-ash particles are concentrated mostly in the range of some microns and formed mainly by SiO₂, CaO, Al₂O₃, MgO, K₂O and Fe₂O₃.
  - Char particles are formed mainly by C.
Study of effects of particles on radiative heat transfer in biomass boilers

• Determining types and amounts of particles.

• Prediction of non-gray radiative properties and accurate phase functions, and

• Evaluation of particle effects on radiative heat transfer by designated test cases.
On Solution of the RTE

• Different radiation models will give different results
• Some radiation models are better suited for different situations, see next table
• Good if solution method of RTE is compatible with that of other equations
• Be able to handle scattering (due to soot agglomerates)
• Be able to handle spectral calculations
• Be able to handle complex geometries
Comparison of Methods for Solving the RTE

<table>
<thead>
<tr>
<th>Method</th>
<th>Angular resolution</th>
<th>Spatial resolution</th>
<th>Spectral resolution</th>
<th>Scattering medium, 2D</th>
<th>Scattering medium, 3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTM</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>1</td>
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<tr>
<td>DOM</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
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<td>Spherical harmonics</td>
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<tr>
<td>Zonal method</td>
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<td>2</td>
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<tr>
<td>Monte Carlo</td>
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<td>4</td>
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<tr>
<td>Finite element/volume techniques</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

4 = very good, 3 = good, 2 = acceptable, 1 = not good
Applications of Radiative Heat Transfer with Participating Media
Numerical Simulations in a Small Furnace

- Furnace model

- Radiation heat transfer and radiative properties are based on temperature and species fields.
• Volume flow rate of air 31.6 m³/h
• Volume flow rate of kerosene 6.6 litre/h
• DOM (the so-called S₈ model was used)
• SNB for CO₂ and H₂O
The flow in the furnace is very non-homogeneous and complex as it passes the swirler.
Numerical Simulation in a Small Furnace

• Soot radiation contribution

(a) With scattering

(b) Without scattering

Participating media are gases H2O, CO2, CO and soot
Numerical Simulation in a Small Furnace

- Soot radiation contribution

(a) Radiative Heat Flux

(b) Radiation Contribution

- Soot has a big influence on the radiative heat flux
- Scattering has no influence on the radiative heat flux
Test facility at B & W Völund Esbjerg, Denmark
Figure 2 (a), (b). Left figure (a) shows the Gunners radiometer. Right figure (b) shows two Gunners radiometers mounted on a furnace wall.
Figure 3 (a), (b). Left figure (a) shows the nozzle of the suction pyrometer. Right figure (b) shows the suction pyrometer inside the furnace close to the grate.
Pellets in the tests
Gas temperature distribution central plane
Furnace wall irradiation
EFG (externally fired gas turbine) Cycle Efficiency and a Proposal for Increasing Its Efficiency

The EFG cycle efficiency

$$\eta = \frac{\dot{W}}{Q} = 1 - \frac{T_1}{T_3} (PR)^{\frac{k-1}{k}}$$

As a solution for increasing turbine inlet temperature (TIT), a duct around the combustor has been proposed which can be empty or filled with some porous materials.
Case Study for Evaluation of a New Concept of EFGT

A cylindrical combustor with the following conditions:

- **Duct side:** 3 atm, 923 K and 0.1 kg/s
- **Combustor:** 1 atm, 710 K and 0.1 kg/s
Combustion Analysis

Three processes, drying, pyrolysis and char combustion, are simplified in a combustion model.

**Input:** Air temperature and theoretical air percentage

**Output:** Temperature and mole fractions of species at the bed outlet and flame temperature.

- **Over the bed (flame) zone:** Combustion of pyrolysis gases
- **Bed zone:** Drying, pyrolysis and char combustion products (Tar, CH₄, CO, CO₂, H₂O, H₂, N₂)
- **Air (turbine exhaust):** Drying of wood pellets, pyrolysis, char combustion
Porous Duct Results

In the similar case with an empty duct, by keeping $D_d-D_c = 0.050$ m constant, $d_p = 0.0075$ m, $\varepsilon = 0.426$.

The pressure drop calculated by Forchheimer extension of the Darcy model which is function of particle diameter and porosity of the porous media.

Heat transfer by radiation and convection from the combustor to the porous duct increased up to 16 kW.
Porous Duct Results

Effects of different particle diameters on the duct outlet temperature ($T_o$) and pressure drop (DP), by selection of different sizes have been investigated. ($D_d - D_c = 0.050$ m is constant).
Gas Turbine Combustor-Why Liner Cooling Is Needed?

Thermal tolerances of materials and extremely long operating intervals of combustors are the main reasons for need of cooling. These problems often make buckling and cracking in liners.

VT4400 Cooled and Non-cooled Cases

Source: Volvo Aero Corporation now GKN Aerospace Engine Systems
A Conjugate Heat Transfer Model for Prediction of Temperature and Heat Loads in Combustors

A lean pre-mixed combustor, with two different simple convective cooling and ribbed cooling arrangements with TBC (thermal barrier coating) have been modeled by CFD and conjugate heat transfer approach. The general combustor data are:

- Swirl number = 0.6
- Liner Conductivity = 25 W/mK
- TBC Conductivity = 1.3 W/mK
Model Description

- Using a 2D Axi-Symmetric geometry
- Simplification of the Complex Swirl system
- Using Multiblock Structured grid
  - 42580 Cells for Simple Cooling Duct
  - 70090 Cells for Ribbed Cooling Duct and TBC
Solution Method and Boundary Conditions (BC)

1) Flow Field

- Solving the Time Averaged Continuity and Navier-Stokes Equations.
- Using SIMPLE Algorithm for the Pressure and Velocity Couplings.
- Assume that the Density Is Temperature Dependent.
- Solving the Transport Equations for the Turbulent Kinetic Energy and Turbulent Dissipation, Using the Standard $k$-$\varepsilon$ Model.
- Using Inlet and Pressure BCs for Inlets and Outlets and Periodic and Symmetry BCs on the $r$-$z$ Faces in the Liner and Cooling Duct, respectively.
Solution Method and Boundary Conditions (BC)
2) Energy Equation

- Using the Enthalpy Form of Energy Equation.
- Solving the Energy Equation in Both Fluid and Solid Domains.
- Involving The Radiative Heat Transfer As Source Terms of Energy Equation.
- Modeling of Combustion By Assumption of One Step Burning of Methane and Using The Eddy Dissipation Concept (EDC) for estimating the reaction rate.
- Using Polynomial Correlation of Cp versus Temperature.
Solution Method and Boundary Conditions (BC)

3) Heat Transfer

- Using the Standard Wall Function Approach on Both Hot and Cold Sides.

- Modeling Radiative Heat Transfer on the Hot Side by:
  - The S4 – Discrete Ordinates Method with 24 Directions.
  - The Spectral Line Weighted Sum of Grey Gases (SLW), With 5 Optimised Gray Gases for Modeling of CO₂ and H₂O Mixture.
Other Solution Considerations

The Computational Fluid Dynamics Code STAR-CD now STAR CCM+ was used.

The Second Order Monotone Advection and Reconstruction Scheme (MARS) was used.

Convergence criteria were set on 1.0 e-4 for normalized global residuals and besides the temperature data at some boundaries were controlled.
Results of Flow and Temperature Fields

- Temperature Field Near to the Ribbed Liner Wall
- Velocity Field Near to the Ribbed Liner Wall
- Temperature Field in the Ribbed Wall
- Temperature Field in the Simple Wall
Results of Flow Field and Temperature in the Liner (Simple Cooling Duct Without Radiation)

Zero Axial Velocity at the Liner Wall

Variation of Temperature Near the Liner Wall

Zero Axial Velocity at the Liner Wall
Results of Temperature Field in the Ribbed Cooling Duct and TBC

Notes:
- Because of extended heat transfer surfaces, the temperature varies along the liner length.
- The average temperature data without radiation increased about 40 K by adding the radiative heat transfer.
- Good agreement has been found in the mid part and also there are some uncertainties about two end points of the experimental data.
Results of Predicted Heat Loads

Notes:

- Radiation increases the amount of heat load more in the cold zones.
- Average heat load by radiation has increased by 8 and 7 percent in the simple and ribbed cooling ducts, respectively.
- Nearly constant heat load at mid part of the simple cooling duct has been predicted.
Some relevant literature


A Catalog of Radiation Heat Transfer Configuration Factors available at:
http://www.me.utexas.edu/~howell/index.html