LUNDS TEKNISKA HÖGSKOLA
ENERGIVETENSKAPER/STRÖMNINGSTEKNIK
Turbulence – theory and modelling (MVK 140)
Exercise 1

Governing Equations and Averaging

1 The conservation equations of mass and momentum can be written as:

$$\frac{\partial u_i}{\partial x_i} = 0$$
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial x_j}$$

What assumptions have to be made to obtain these equations? Think of examples where the assumptions are questionable.

2 Show that time averaging has the following properties:
$\overline{\overline{u}} = \overline{u}$
$\overline{u'} = 0$
$\overline{u+v} = \overline{u} + \overline{v}$
$\overline{uv} = \overline{u}\ \overline{v} + \overline{u'v'}$
$\overline{\frac{\partial u}{\partial x}} = \frac{\partial \overline{u}}{\partial x}$

3 Show using Reynolds decomposition (assume taking the mean and taking the derivative commute):

a $\frac{\partial \overline{u_i}}{\partial x_i} = 0$

b $\frac{\partial u_i'}{\partial x_i} = 0$

4 a Derive the transport equations for the velocity fluctuations (subtract the average from the instantaneous momentum equations).
$\frac{\partial u_i'}{\partial t} + \overline{u_j}\frac{\partial u_i'}{\partial x_j} + u_j'\frac{\partial \overline{u_i}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p'}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial u_i'}{\partial x_j} - u_i'u_j' + \overline{u_i'u_j'}\right)$

b Take the mean of the velocity fluctuation transport equations.

5 What is meant by the closure problem of the equations governing turbulent flows?

Turbulence and turbulence modelling

6 You observe the flow in a small river. Determine by using the phenomenological criteria for turbulence if the flow can be regarded as turbulent.

7 Consider flow of water ($\nu = 10^{-6}m^2/s$) between two parallel plates. The bulk flow velocity is $U = 0.1m/s$. The distance between the plates is $L = 0.1m$.

a Estimate times scales of diffusion and convection. Show that their ratio is the Reynolds number. Is the flow turbulent?

b What are the time scales if the distance is $L = 10\mu m$ instead? Is the flow turbulent?

8 The earth rotates at a rate $\omega_{earth} = \frac{2\pi}{day}$. A characteristic time scale is therefore $T_{earth} \sim 10^4 s$. Estimate a time scale for a hurricane and a time scale for a bath tub vortex. Do you expect rotational bias in each case? What determines the direction of rotation? Is there any difference if the hurricane/bath tub is located in Lund ($\approx 55^o N$) or Port of Spain ($\approx 10^o N$). Hint: think about the Rossby number $Ro = \frac{U}{2 \cdot L \cdot \omega sin(\phi)}$.

9 With a measurement method you are able to generate 2D pictures of the instantaneous velocity field in a transparent pipe with a diameter of 10 cm. The bulk velocity in the pipe is 10 m/s and you expect a turbulence intensity of 5%. Since your hard disk is getting full you can save only a limited number of pictures.

a Assess the accuracy of the estimated average bulk flow velocity if 11 samples are collected.

b Repeat the same exercise assuming that 51 samples were used for the statistics.

c How would you asses the accuracy of the rms velocity fluctuations measured?

d Assuming normal distributions, estimate the minimum number of images needed to assure a confidence interval of maximum 0.01 m/s with a confidence of 95% accuracy.

e Assuming that you have a fast enough measurement system, how often are you going to sample?

Mean squared error of statistical quantities:

	Formula	Mean Squared Error (MSE)
Average	$\bar{x} = \frac{1}{n}\Sigma x_i$	$MSE(\bar{x}) = E((\bar{x}-\mu)^2) = \left(\frac{\sigma}{\sqrt{n}}\right)^2$
Population variance	$S_n^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$	$MSE(S_n^2) = E((S_n^2 - \sigma^2)^2) = \frac{2n-1}{n^2}\sigma^4$

Coefficients for confidence intervals:

t	80%	90%	95%
Normal	1.281	1.644	1.959
Stud 10	1.372	1.812	2.228
25	1.316	1.708	2.060
50	1.299	1.676	2.009
∞	1.282	1.645	1.960

Stability and Transition

10 What is meant by critical Reynoldsnumber (Re_c)? How is Re_c affected by external disturbances?

11 The Orr-Sommerfelds equation can be written as
$(U-c)\left(\frac{d^2\hat{v}}{dy^2} - k^2\hat{v}\right) - \hat{v}U'' - \frac{1}{i\alpha Re}\left(\frac{d^2}{dy^2} - k^2\right)^2\hat{v} = 0$
where $Re = \frac{U_0 d}{\nu}$ is the Reynolds number and $c = c_r + ic_i$ is the phase velocity.

  a Derive, starting from the above equation, the Rayleigh instability equation.

  b After some manipulation the Rayleigh instability equation becomes
  $c_i \int_0^1 \frac{U''|\hat{v}|^2}{(U-c_r)^2 + c_i^2} dy = 0$
  Show, using this expression, that a point of inflection in the velocity profile is an necessary condition for inviscid instability.

12 Consider flow over a flat plate. How is the stability, for this kind of flow, affected by an adverse pressure gradient? Motivate!

13 Consider flow over a concave surface.

  a Which term in the momentum equations balances the centrifugal force?

  b Is this flow stable? Motivate!

  c Is the stability affected if the surface is convex instead? Motivate!